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Boundary effects on car accidents in a cellular automaton model

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Abstract

In this paper we numerically study the probability P_{ac} of occurrence of car accidents in the Nagel–Schreckenberg (NS) model with open boundary condition. In the deterministic NS model, numerical results show that there exists a critical value of extinction rate β above which no car accidents occur, and below which the probability P_{ac} is independent of the speed limit v_{max} and the injection rate α, but only determined by the extinction rate β. In the non-deterministic NS model, the probability P_{ac} is a non-monotonic function of β in the region of low β value, while it is independent of β in the region of high β value. The stochastic braking not only reduces the occurrence of car accidents, but splits degenerate effects of v_{max} on the probability P_{ac}. Theoretical analyses give an agreement with numerical results in the deterministic NS model and in the non-deterministic NS model with $v_{max} = 1$ in the case of low β value region. Qualitative differences between open and periodic systems in the relations of P_{ac} to the bulk density ρ imply that various correlations may exist between the two systems.

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1. Introduction

Recently, traffic problems have attracted much attention from scientists. Traffic flow is a kind of many-body system of strongly interacting cars which cannot be described in the framework of standard equilibrium statistical mechanics. Recent studies revealed complex physical phenomena among which are hysteresis, synchronization of flow, wide moving jams and phase transitions [1, 2]. On the other hand, traffic jams and traffic accidents have become significant problems in a modern society. Jams often appear in densely populated areas where the capacity of the existing traffic network is exceeded. Cars in jams may emit considerable amounts of toxic gases which are harmful to the environment and human health. Therefore, many modelling approaches have been proposed to simulate traffic systems and to design on-line controls for efficient traffic optimization [3].

Most recently, the occurrence of car accidents has been studied within the framework of cellular automaton (CA) models with periodic boundary conditions. With the help of the conditions for the occurrence of car accidents first proposed by Boccara *et al* [4], simulations of the probability for car accidents to occur have been presented in the NS model and the Fukui–Ishibashi (FI) model [5]. And analytical expressions for car accidents have also been provided in the model with $v_{\max} = 1$ in the case of $p \neq 0$ [6], and without considering stochastic braking [7], respectively. The relations of car accidents to traffic flow and stopped cars in the periodic system have also been studied by us [8]. However, the probability for car accidents to occur in an open traffic system has not been discussed yet.

The most significant difference between systems with open and periodic boundary conditions is car density. In a system with periodic boundary conditions, car density is considered an adjustable parameter. However, in the case of open boundary condition, a traffic system has two adjustable parameters, namely, the injection rate and the extinction rate which characterize a car moving in and out of the system respectively, and the car density in the system is only a derived parameter. Moreover, compared with periodic systems, numerical studies imply that open systems show different behaviour of quantities such as global density, current, density profile and even microscopic structure of jammed phases [9–15]. Therefore, these differences give us an impetus to study the probability of occurrence of car accidents in more realistic open traffic systems.

In this paper, we do not really study car accidents. We numerically investigate the probability of occurrence of dangerous situations in which careless drivers might cause accidents, within the CA model with open boundary conditions, including deterministic and non-deterministic cases. According to the three necessary conditions for the occurrence of car accidents [4] and the results of [8], the probability of occurrence of car accidents is directly related to traffic flow and stopped cars; therefore, the studies on the probability can lead us to understand traffic flow. Secondly, if accidents really occur, the ‘wrecked’ cars can interrupt traffic. In this case, car accidents can reduce traffic flow, but traffic flow is directly related to car accidents. This complex situation should be further investigated. The paper is organized as follows. Section 2 is devoted to the description of the model and the conditions for the occurrence of accidents. In section 3, numerical studies of car accidents are given, and the effects of stochastic braking and speed limits are considered. A phenomenological theory is also presented to describe the computer simulations. Finally, the results are summarized in section 4.

2. Model and car accidents

Our studies are based on a one-dimensional cellular automaton model introduced by Nagel and Schreckenberg [16]. The road is divided into L cells of equal size numbered $i = 1, 2, \ldots, L$ and the time is discrete. Each site can be either empty or occupied by a car with the integer speed $v = 0, 1, 2, \ldots v_{\max}$, where $v_{\max}$ is the speed limit. The speed v is mainly determined by the distance from the car ahead. When the distance increases, the car accelerates; when the distance decreases, the car slows down. Let d denote the distance from the car ahead. At each time step, the following four steps are performed simultaneously for all cars:

(1) acceleration: increase v by 1 if $v < v_{\max}$.
(2) slowing down: decrease v to d if $v > d$.
(3) stochastic braking: decrease v by unity with probability p if $v > 0$.
(4) movement: move the car v sites forward.

The acceleration under the speed limit and the slowing down due to the car ahead are prescribed by the first two rules. The model parameter p in the third rule describes individual velocity fluctuations due to delayed acceleration. Without the third rule, the model is deterministic. Iterations over these simple rules already give realistic results.

Recent studies have shown the way in which open boundary conditions are implemented to alter the behaviour of the model [18, 19]. However in the present paper, we mainly study the probability for car accidents to occur; therefore, open boundary conditions are defined in the following way [13, 14], in which the model shows the same relationship between traffic flow and bulk density as in the case of periodic boundary conditions, except that the phase of maximum flow is absent [12]. At site $i = 0$ which means out of the system, a car with speed $v = v_{\max}$ is created with probability α. The car immediately moves forward in accordance with the NS rule. If the site $i = 1$ is occupied by a car, the injected car at site $i = 0$ is deleted. At $i = L + 1$ a 'block' occurs with probability $1 - \beta$ and causes a slowing down of the cars at the end of the system. Otherwise, with probability β, the car simply moves out of the system. The model with open boundary conditions has four basic controlling parameters among which are the speed limit $v_{\max}$, the stochastic braking probability p, the injection rate α and the extinction rate β.

In the basic NS model, car accidents will not occur, because the second rule of the update is designed to avoid accidents. The safety distance of drivers is respected in the driving scheme. However, in real traffic car accidents occur most likely due to careless driving of the drivers who do not maintain the safety distance. More precisely, if the car ahead is moving, expecting it to be moving at the next time step, a careless driver has a tendency to drive as fast as possible and increases the safety velocity given in the second rule of update by one unit with a probability p'. At the next time step, it will arrive at the position of the moving car ahead. If the moving car ahead suddenly stops, collision between the two cars takes place. The necessary conditions for determining the occurrence of car accidents which have been proposed by Boccara *et al* read as follows [4]. The first condition is that $d \leqslant v_{\max}$, which means that a car will arrive at the site of the car ahead by the next time step. The second condition is that a car ahead is moving. And the third is that the moving car ahead suddenly stops at the next time step.

Without considering stochastic braking in the periodic system, the velocity of a car is equal to the number of empty cells in front of it when the car density is larger than the critical density $\frac{1}{1+v_{\max}}$; therefore, when the three necessary conditions for the occurrence of car accidents are simultaneously satisfied, the cars following can reach the positions of the stopped cars ahead and car collisions occur if the velocity of the cars following increases by one unit. In the presence of stochastic braking, even if the number of empty cells between two neighbouring cars is less than the speed limit $v_{\max}$, with the increase in safety velocity given in the second update rule, the cars following fail to reach the position of the stopped cars ahead, because of being randomly delayed, while they must collide with the stopped cars ahead when braking is not considered. Thus, to correctly determine car accidents caused by careless drivers, we propose modified accident conditions which are applicable for both deterministic and non-deterministic systems [17]. The first condition is that over the iterations of rules (1)–(3), the velocity of a car is exactly equal to the number of empty cells in front of it, which means that the car can reach the position of the car ahead if the velocity of the car driven by the careless driver increases by one unit. The second condition is there is a moving

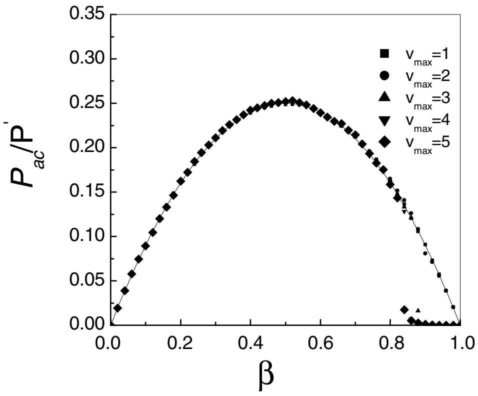

Figure 1. Probability $P_{\rm ac}$ (scaled by p') as a function of extinction rate β in the deterministic NS model for the case $\alpha = 1$. Solid lines correspond to analytical results, and symbol data are obtained from numerical simulations.

car ahead. The third condition is that the moving car ahead suddenly stops. If the above three conditions are satisfied simultaneously and the velocity of the cars following increases by unity with probability p', collision between two cars will occur. These three necessary conditions for car accidents are also applicable for the open system.

When the three necessary conditions are met simultaneously, dangerous situations for the occurrence of car accidents may arise. If the velocity of the following cars driven by careless drivers increases by one unit with probability p', a car accident will take place. Obviously, the probability of car accidents is proportional to the occurrence of dangerous situations, the proportional constant being p'. Usually, the probability per car per time step for car accidents to occur is denoted by $P_{\rm ac}$. Apparently, the probability $P_{\rm ac}$ is proportional to the probability p'; therefore, the probability p' can be considered a scaling parameter. In the process of simulations, car accidents do not really occur. These dangerous situations are calculated and considered as a signal for the occurrence of accidents when the three necessary conditions are simultaneously satisfied. Here, the system size $L = 1024$ is selected and the results are obtained by taking averages over 25 initial configurations and 2×10^4 time steps after discarding 1×10^5 initial time steps for each configuration.

3. Numerical results and theoretical analysis

3.1. The deterministic case

We investigate the influence of the boundary on the probability $P_{\rm ac}$ for the injection rate $\alpha = 1$. Firstly, the stochastic braking of drivers is not considered, i.e., $p = 0$. Figure 1 shows the accident probability $P_{\rm ac}$ as a function of β for various values of $v_{\rm max}$. As shown in figure 1, with the increase in β, the probability increases, reaches a maximum but decreases with further increase in β. Near $\beta = 0$, the value of the probability $P_{\rm ac}$ increases linearly with the increase in β. However, at a very high extinction rate, a linear drop in the behaviour of the probability with increasing β is observed in figure 1. According to the first condition for the occurrence of car accidents, the probability $P_{\rm ac}$ is related to traffic flow. In the case of low extinction rate β, the stopped cars assume a large value and the value of traffic flow is very small; therefore, the probability $P_{\rm ac}$ is determined from the value of β linearly. As described in the third condition for car accidents, $P_{\rm ac}$ is related directly to the suddenly stopped cars, and the number of the

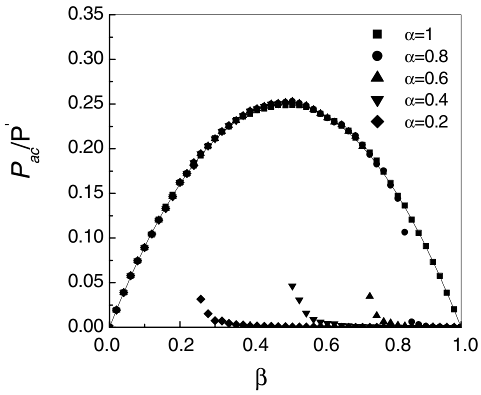

Figure 2. Probability P_{ac} (scaled by p') as a function of extinction rate β in the deterministic NS model with $v_{max} = 1$ for various values of injection rate. Solid lines are analytic results, and symbol data are obtained from numerical simulations.

stopped cars is very small in the case of a high value of β; therefore, the value of P_{ac} decreases linearly with increasing β.

The most remarkable results of our investigation show that the probability P_{ac} is independent of the speed limit v_{max}. In figure 1, the probability for car accidents to occur in the case of $v_{max} = 2$ shows similar behaviour to that in the case of $v_{max} = 1$. But in the case of $v_{max} \geqslant 3$, there is a critical value of the extinction rate β_{cv}, below which P_{ac} is independent of the speed limit and shows a scaling behaviour, while above which P_{ac} rapidly decreases to zero. The position of β_{cv} shifts towards a low value of β with increasing speed limit v_{max}. In accordance with previously reported results in [13], we know that the state of a system is determined by the extinction rate β. In the case of $v_{max} < 3$, the state of a system with injection rate $\alpha = 1$ is in the jamming phase, while in the case of $v_{max} \geqslant 3$, the jamming state exists in the region of low value of β, and free flow lies in the region of high value of β. Let β_{cv} denote the critical rate. Above β_{cv}, there are no car accidents, because there are no stopped cars in the free-flow phase of the system. Below β_{cv}, the systems are involved in the jamming states; therefore, car accidents occur. In the jamming states, the extinction rate β controls the motion of cars; thus the probability P_{ac} is independent of the speed limit and is controlled by the extinction rate β.

For $\alpha < 1$, a discontinuous change in the probability P_{ac} occurs at a critical point $\beta_{c\alpha}$ above which the value of P_{ac} decreases abruptly to zero. Below $\beta_{c\alpha}$, the values of the probability P_{ac} for various values of v_{max} collapse into a single curve. The results are shown in figure 2. The position of the transition point $\beta_{c\alpha}$ shifts towards a low value of β with decreasing injection rate α. According to reports in [11, 13], traffic states in the NS model with open boundary conditions present a phase transition from a low-density phase to a high-density phase with increasing β at the transition point $\beta_{c\alpha}$ for the case of $\alpha < 1$. Above $\beta_{c\alpha}$, there are no stopped cars, and therefore no car accidents occur. Below $\beta_{c\alpha}$, the system exhibits the jamming state in which the sudden stoppage of cars results in the occurrence of car accidents.

In the low-density phase, no stopped cars exist and therefore no car accidents occur. However, in the high-density phase, the probability for the occurrence of car accidents is controlled by the extinction rate β and is independent of the speed limit v_{max}, as shown in figures 1 and 2. In the case of large bulk densities, i.e., large values of α and $\beta \ll 1$, car accidents frequently occur at the right boundary. According to the definition of extinction

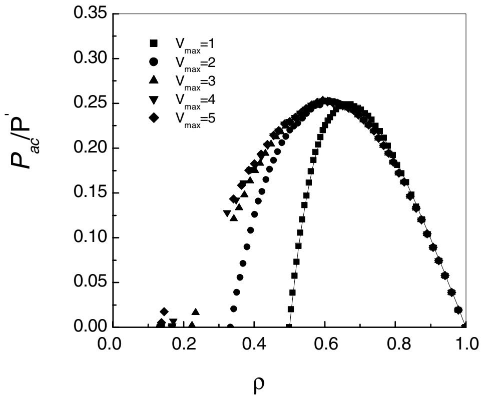

Figure 3. The relation of probability P_{ac} (scaled by p') to the bulk density ρ in the deterministic NS model. Symbol data are obtained from computer simulations, and the solid line corresponds to analytic results. Some values of the probability P_{ac} corresponding to the case $v_{\text{max}} > 2$ cannot be observed in certain regions of density.

rate β, the mean waiting time of the last car at site L is $T = 1/\beta$ [11]. On the other hand, the last car moves and occupies the last site with probability $1 - \beta$ if the safety velocity of the car increases. According to the definition of the probability P_{ac}, we find that for large α and $\beta \ll 1$, the probability P_{ac} reads

$$P_{\text{ac}} = p'(1-\beta)/T = p'\beta(1-\beta). \tag{1}$$

In formula (1), the probability P_{ac} is directly related to the probability for a car moving out of the system or the probability for a car occupying the last site of the system. The product of two kinds of probabilities means the probability per car for two cars occupying the same cell simultaneously. As shown in figure 1, theoretical analysis is in good agreement with numerical results.

Although car density is only a derived parameter and controlled by the injection rate α and the extinction rate β in open systems, the relations of the probability P_{ac} to the average car density ρ are similar to those for periodic systems in which car density is an adjustable parameter. The results are shown in figure 3. We can see in figure 3 that there is a critical density below which no car accident takes place, and above which the probability P_{ac} is a non-monotonic function of the average car density ρ. In the high-density region, a scaling relation is also observed and the probability P_{ac} decreases linearly with the increase in the average density. The maximum probability shifts towards the low-density region with the increase in the speed limit. Especially for the cases $v_{\text{max}} = 1$ and 2, the probability P_{ac} is similar to that in systems with periodic boundary conditions. But for $v_{\text{max}} > 2$, as shown in figure 3, some values of the probability P_{ac} can never be observed in certain regions of density because the values of the bulk density ρ controlled by extinction rate β cannot be derived [12, 13].

Quantitatively, the probability P_{ac} in an open system is different from that in a periodic system, especially in the case of $v_{\text{max}} = 1$. In fact, an explicit expression for the relation of the accident probability to the bulk density in the deterministic NS model with $v_{\text{max}} = 1$ can easily be obtained. Let ρ denote the bulk density of cars. According to [9], in the limiting case $L \to \infty$, the density in the high-density phase $\alpha > \beta$ can be written as

$$\rho = \frac{1}{1+\beta}. \tag{2}$$

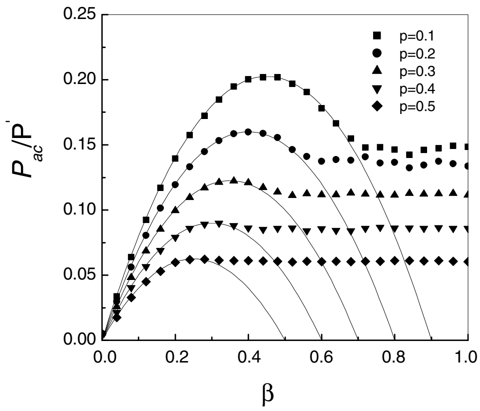

Figure 4. Probability P_{ac} (scaled by p') as a function of extinction rate β in the non-deterministic NS model with $v_{max} = 1$. Solid lines are obtained from explicit expression (4), and symbol data are numerical results. The case $\alpha = 1$ is considered.

Substituting (2) into (1), we find that the probability P_{ac} obeys the following relation:

$$P_{ac} = p' \frac{(1-\rho)(2\rho - 1)}{\rho^2} \qquad \text{for} \quad \rho \geqslant 0.5. \tag{3}$$

As shown in figure 3, theoretical results are in good agreement with numerical data in the case of $v_{max} = 1$. Since the necessary conditions for car accidents to occur concern states of two consecutive cars at different time steps, the phenomena of car accidents involve spatial and temporal correlations, and therefore any success of the phenomenological mean-field theory is encouraging.

Compared with results for periodic systems, the additional factor $\frac{1}{\rho}$ in equation (3) has never been achieved as the braking probability decreases to zero in periodic systems [6]. In the NS model with $v_{max} = 1$, the maximum distance between a cell occupied by a suddenly stopped car and a cell where a car driven by a careless driver will collide with the car ahead at the next time step if the safety velocity increases by one unit must be unity. Thus, the mean correlative length between flow and stopped cars in the process of car accidents is $\frac{1}{\rho}$. But such a correlation no longer exists in periodic systems in the case of $p = 0$. Therefore, we conclude that the boundary can induce some correlations in the process of car accidents, although the relationship between traffic flow and bulk density ρ is the same as that in the case of periodic boundary conditions [12].

3.2. The non-deterministic cases

Next, we study the probability for the occurrence of car accidents when the stochastic braking behaviour of drivers is considered, i.e., $p \neq 0$. Figure 4 shows the relation of probability P_{ac} to the extinction rate β in the case of $v_{max} = 1$. With increasing β, the value of P_{ac} increases, reaches a maximum and then decreases with further increase in β. However, when the value of β is larger than a critical value β_{cp}, the probability P_{ac} is independent of β. With increasing randomization probability p, the probability P_{ac} is suppressed, and thus the position of β for the maximum of P_{ac} where car accidents occur most frequently shifts towards the low-value region of β.

Similar to the previous study of the occurrence of car accidents [8], the probability P_{ac} is related not only to the stopped cars, but also to traffic flow. For the case $v_{max} = 1$, any braking

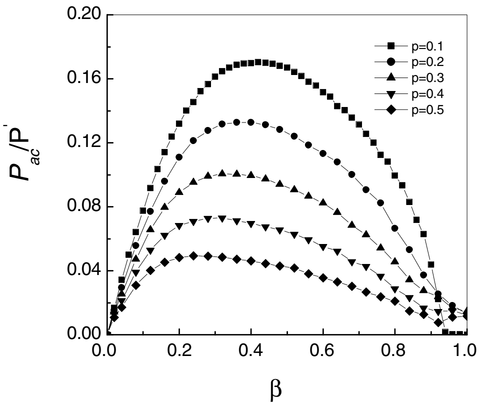

Figure 5. Effects of stochastic braking on probability P_{ac} (scaled by p') in the non-deterministic NS model with $v_{\max} = 5$. The case $\alpha = 1$ is considered.

behaviour can cause the occurrence of stopped cars, therefore resulting in car accidents. With the increase in the braking probability p, traffic flow decreases, and accordingly the probability P_{ac} is decreased. In the low-value region of β, traffic flow is very small, and therefore the probability P_{ac} increases linearly with increasing β. As the value of β increases, and when it is larger than a critical value β_{cp} with $\beta_{\mathrm{cp}} = 1 - \sqrt{p}$, the system is in the maximum current phase where traffic flow is independent of the extinction rate β and is only dependent on the braking probability p [9], and therefore the accident probability P_{ac} is independent of β.

When the extinction rate β is lower than β_{cp}, the traffic systems exhibit a jamming state. In the case of large bulk density, i.e., large values of α and $\beta \ll 1$, car accidents often occur at the right boundary. The extinction rate β means the probability for the last car to leave the system, and thus the average waiting time for the last car at the last site is $T = 1/\beta$ [11]. During the time interval, the last site is occupied again by a car with probability $1 - p - \beta$ if the safety velocity of the car increases. Thus, according to the definition of the occurrence of car accidents, the probability P_{ac} can be written as

$$P_{\mathrm{ac}} = p'(1 - p - \beta)/T = p'(q - \beta)\beta \tag{4}$$

where $q = 1 - p$. Expression (4) shows that the accident probability P_{ac} is controlled by the extinction rate β and the probability of stochastic braking. Comparison of our prediction for the probability P_{ac} with computer simulations gives excellent agreement.

We also calculate the probability P_{ac} in the case of $v_{\max} = 2$. According to previous studies of the phase diagram for $v_{\max} = 2$ [14], the phase diagram shows strong similarities to the $v_{\max} = 1$ case, which include three states—free flow, jamming and maximum current state—and, along the line $\alpha = 1$, exhibits only a phase transition from the jamming state to the maximum current state with increasing β. Therefore, the probability P_{ac} for the case $v_{\max} = 2$ shows similarities to the case $v_{\max} = 1$ (these numerical results are not shown).

However, for $v_{\max} > 2$, the probability P_{ac} is different from that for the case $v_{\max} = 1$, due to variation of phase. Figure 5 shows the relation of the probability P_{ac} to the extinction rate β in the case of $v_{\max} = 5$. With increasing braking probability p, the probability P_{ac} will decrease, especially for low values of β. However, in the region of high values of β, the characteristic relations of the probability P_{ac} to the extinction rate β above cannot be observed. As shown in figure 5, when randomization probability p is smaller than a critical value $p_c = 0.12$, no car accidents take place, because the system is in the free moving phase.

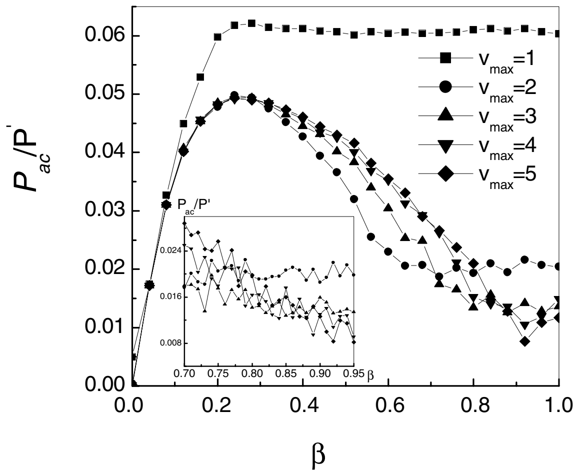

Figure 6. Effects of speed limit on probability $P_{\rm ac}$ (scaled by p') in the NS model in the case $p = 0.5$. The case $\alpha = 1$ is considered. The inset shows the relation of $P_{\rm ac}$ to β in the region near $\beta = 0.75$ in the case of $v_{\rm max} > 1$.

But when $p > p_c$, the maximum current phase of the systems is represented; thus, the value of $P_{\rm ac}$ tends to be invariable.

The stochastic braking p not only reduces the probability $P_{\rm ac}$, but also splits the degenerate effects of the speed limit on the probability $P_{\rm ac}$. The relation of $P_{\rm ac}$ to the extinction rate β for different values of $v_{\rm max}$ is shown in figure 6. As shown in figure 6, when the value of β is very small, the probability $P_{\rm ac}$ shows a scaling relation and is independent of the speed limit. In this case, the extinction rate β determines whether or not car accidents occur, due to small traffic flow. However, when β is larger, different values of $v_{\rm max}$ correspond to those of $P_{\rm ac}$ in the case of stochastic braking. In the region of large value of β, $P_{\rm ac}$ decreases with the increase in $v_{\rm max}$, as shown in the inset in figure 6. However, between small and large values of β, $P_{\rm ac}$ increases with increasing speed limit. An exception is the case $v_{\rm max} = 1$ in which the values of $P_{\rm ac}$ are larger than those in the case of $v_{\rm max} > 1$ and unchanged in the region of large values of β. It is difficult to understand the relation of $P_{\rm ac}$ to $v_{\rm max}$. In principle, formula (4) can be applied to the case $v_{\rm max} > 1$ if the probability of a car to move from and occupy the last site is known. But, in the non-deterministic NS model with $v_{\rm max} > 1$, the value of the passing rate is not equal to the extinction rate because of extended hopping effects.

4. Summary

In this paper, we study the probability for the occurrence of car accidents in the NS model with open boundary conditions. Different from periodic systems, open systems in which the car density is only a derived parameter are controlled by the injection rate and the extinction rate. Because real traffic systems are usually open, it is highly desirable to investigate the occurrence of car accidents in traffic systems both numerically and theoretically.

Numerical results show that the accident probability $P_{\rm ac}$ in the case of $p = 0$ is independent of the speed limit $v_{\rm max}$ and is controlled only by extinction rate β when extinction rate β is smaller than a critical value $\beta_{\rm cv}$. Above $\beta_{\rm cv}$, no car accident occurs. However, when stochastic braking is considered, the probability $P_{\rm ac}$ decreases with increasing braking probability p in the region of low value of β, while in the region of large values of β, the value of $P_{\rm ac}$ tends to be invariable. Moreover, the feature that the probability P_{ac} is only controlled by β gives us a hint of how to effectively avoid the occurrence of car accidents in open traffic systems.

Stochastic braking splits degenerate effects of the speed limit on the probability P_{ac} which collapse into one line for various values of v_{max}. In the region of small values of β, the value of the probability P_{ac} is determined by the extinction rate β and is independent of v_{max}. In the region of large values of β, P_{ac} decreases with increasing v_{max}. However, between small and large values of β, P_{ac} increases with increasing v_{max}.

The relations of accident probability to bulk density in the deterministic NS model show similar behaviour to those in periodic systems, but in the case of $v_{max} = 1$, quantitative differences are observed, which imply different correlations between the two systems in the process of car accidents.

A phenomenological mean-field theory is presented to describe the accident probability P_{ac} in the deterministic NS model. The probability P_{ac} is proportional to the product of the probability for the last car to move out of the system and the probability for a car to occupy the last cell. Theoretical analysis can give results in agreement with numerical results when β is below the critical value β_{cv}. This expression is also applicable for the probability P_{ac} in the non-deterministic NS model with $v_{max} = 1$ when β is smaller than the critical value β_{cp}. But in the non-deterministic NS model with $v_{max} > 1$, explicit expressions which deserve further investigation could not be obtained because of extended hopping effects.

Acknowledgments

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